$\mathbb{Z}_p$-Continuous Visualization and the Morita Gamma Function

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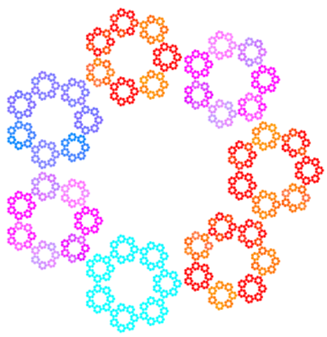
Abstract

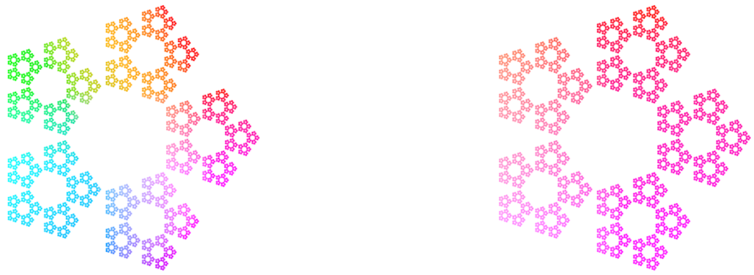
There exist several methods of visualizing the p-adic integers and functions $\mathbb{Z}_p \to \mathbb{Z}_p$. Methods which preserve the coset structure, viewing $\mathbb{Z}_p$ as a Cantor set, tend to fall short with non-polynomial functions or higher primes. In this paper, we introduce a method of visualizing functions on the p-adic integers that works particularly well for $\mathbb{Z}_p$-continuous functions. As a practical example, we use this method to explore the Morita Γ-function, a p-adic analogue of the Γ-function in complex analysis.



1 Two-plane plot of $\mathbb{Z}_p$-continuous functions

Mathematicians such as Dorfsman-Hopkins et al. [2] have illustrated several methods for the visualization of continuous functions $\mathbb{Z}_p \to \mathbb{Z}_p$. Most of these methods utilize a traditional Cantor-set structure to represent the topology of the p-adic integers. Let us consider two common models (also found in [2]), "coset coloring" and "coset displacement." Coset coloring is analogous to the method provided by Harris and Nelson in [2], and for each $n \in \mathbb{N}$ colors the cosets mod $p^n\mathbb{Z}_p$ according to its image's nth digit in a base-p expansion. This method requires the "image's nth digit" to be well-defined, and thus only works for certain, particularly well-behaved maps. Coset displacement is analogous to the experimental method attributed to Dorfsman-Hopkins in [2]. Given a fixed resolution n, this method colors the cosets mod $p^n\mathbb{Z}_p$ uniquely. The user then clicks a button and watches as the cosets are displaced to the position of their (arbitrary) representative's image under the map. This method works well, but falls short in a non-interactive context since information is lost when the map is not injective.

Our method is a flexible, hybrid alternative to the two, and works particularly well for $\mathbb{Z}_p$-continuous functions. As an immediate example, let $f\colon \mathbb{Z}_5 \to \mathbb{Z}_5$ be the function which maps $x \mapsto 5x$ for all $x \in \mathbb{Z}_5$.



(a) $x \mapsto x$ (b) $x \mapsto 5x$

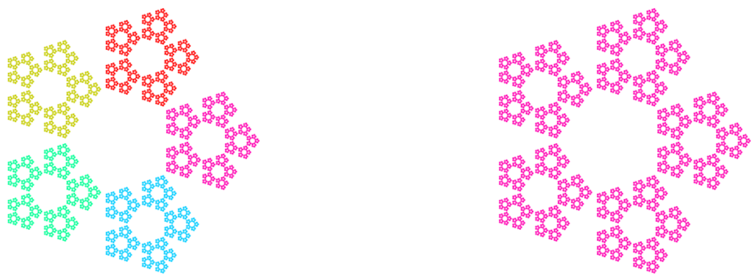
Figure 1: Two-plane plot of $x \mapsto 5x$ in $\mathbb{Z}_5$ using the standard identity plane.

Our graph for f is pictured in Figure 1. Like the coset displacement method, each dot on a plane is a coset of $p^n\mathbb{Z}_p$ for some n (called the *visual resolution* of the plot). The left plane is the *identity plane*, where each coset is assigned a color. The right plane is the *transformation plane*. To color a given coset $k + p^n\mathbb{Z}_p$ on the transformation plane, we see the output of our function on the representative $f(k)$, and assume the color of the coset $f(k) + p^n\mathbb{Z}_p$ on the identity plane.

In our example, the identity plane colors each coset uniquely. Every element of $\mathbb{Z}_p$ is mapped into $5\mathbb{Z}_5$, or the rightmost of the five cosets mod $5\mathbb{Z}_5$. Thus the transformation plane takes the colors of $5\mathbb{Z}_5$, and seems to be a "zoomed in" version of $5\mathbb{Z}_5$ on the identity plane.

The flexibility of this technique comes from our ability to tweak the colors of the identity plane. If we wanted to verify that every output in the transformation plane was in $5\mathbb{Z}_5$ (and not, say, a similarly colored point in $1+5\mathbb{Z}_5$) we can alter the identity plane's colors to highlight this structure. This yields the visualization in Figure 2.

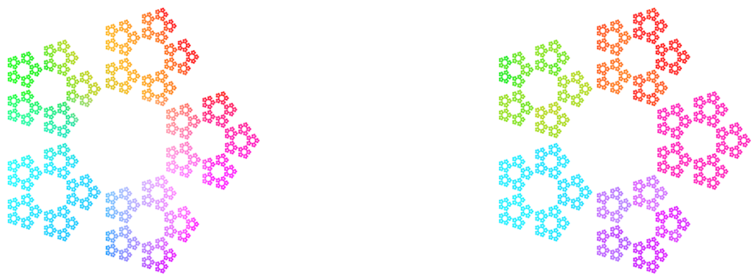
One final note about our visualizations before we explore the Morita gamma function: a continuous function in $\mathbb{Z}_p$ does not necessarily map a visually smooth identity plane



(a) $x \mapsto x$ (b) $x \mapsto 5x$

Figure 2: Two-plane plot of $x \mapsto 5x$ of resolution 5. Elements in the same coset mod $5\mathbb{Z}_5$ are assigned the same color in the identity plane.

to a visually smooth transformation plane. Consider the map $x \mapsto x^5$ on $\mathbb{Z}_5$, which is certainly continuous. With the standard identity plane, we get the plot in Figure 3.



(a) $x \mapsto x$ (b) $x \mapsto x^5$

Figure 3: Two-plane plot of the Frobenius map $x \mapsto x^5$ in $\mathbb{Z}_5$. Notice that the colors in the transformation plane are not smooth between cosets, but still approach uniformity mod higher powers of 5.

The smooth transitions from the cosets mod $5\mathbb{Z}_5$ are lost entirely. All that $\mathbb{Z}_p$-continuity guarantees is that closer elements (elements sharing "deeper" cosets) have more similar colors. Every element in $5\mathbb{Z}_5$ has distance 1 from every element in $1+5\mathbb{Z}_5$, the maximum distance between two numbers in $\mathbb{Z}_5$, and so there is no obligation for elements in these respective cosets to have similar colors.

2 Setting the stage in $\mathbb{C}$

The Γ-function is an invaluable special function in complex analysis. It is a meromorphic extension of the factorial function to the complex plane, and appears frequently in analytic number theory. In this section, we follow Chapter 14 of Gamelin [3] and give a brief tour of the Γ-function and its importance in complex analysis.

Definition 1. The Γ-function is defined on the right half plane

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt$$

for $\Re(z) > 0$.

Let us first convince ourselves that $\Gamma(z)$ extends the factorial function. This is the only complex-analytic result we will prove.

Proposition 1. *For all $n \in \mathbb{N}$, we have*

$$\Gamma(n+1) = n!$$

Proof. Notice we can use integration by parts on $n \in \mathbb{N}$, $n \geq 1$, to obtain

$$\begin{aligned}
\Gamma(n+1) &= \int_0^\infty e^{-t} t^n dt \\
&= \lim_{m\to\infty} \int_0^m e^{-t} t^n dt \\
&= \lim_{m\to\infty} \left[-t^n e^{-t}\right]_0^m + n \int_0^m e^{-t} t^{n-1} \\
&= n\Gamma(n).
\end{aligned}$$

We also have that

$$\begin{aligned}
\Gamma(1) &= \int_0^\infty e^{-t} t^0 dt \\
&= \lim_{m\to\infty} \int_0^m e^{-t} dt \\
&= \lim_{m\to\infty} \left[e^{-t}\right]_0^m \\
&= 1.
\end{aligned}$$

Thus by an inductive argument, we have $\Gamma(n+1) = n!$ for $n \geq 1$. ■

Importantly, Γ extends (except for simple poles) to $\mathbb{C}$.

Theorem 1. *$\Gamma(z)$ extends to be meromorphic on $\mathbb{C}$ with simple poles at $-n$, for $n \in \mathbb{N}$. It satisfies the equation*

$$\Gamma(z+1) = z\Gamma(z) \tag{1}$$

on this domain.

We also have an important identity concerning the Γ-function.

Proposition 2. *Γ satisfies the* Legendre relation

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}$$

for $z \in \mathbb{C} \setminus \mathbb{N}$.

3 The Morita Γ-function

The question arises: can a similar extension of the factorial function (or Γ-function) hold in the p-adics? It turns out no such continuous extension exists. Robert [7] uses a slick approach to prove this. We will follow his direction, although we'll tune our proof to be more direct. We first need to prove that the set of naturals greater than or equal to some lower bound is dense in $\mathbb{Z}_p$

Lemma 1. *The set $\{k \in \mathbb{N} \mid k \geq N\}$ is dense in $\mathbb{Z}_p$ for all $N \in \mathbb{N}$.*

Proof. Let $N \in \mathbb{N}$, $x \in \mathbb{Z}_p$, and $\epsilon > 0$ be arbitrary. It suffices to show that $x \in B(k, \epsilon)$ where $k \geq N$. Fix two natural numbers m_1 and m_2 such that $N \leq p^{m_1}$ and $|p^{m_2}| < \epsilon$. Call $m := \max\{m_1, m_2\}$ and consider the natural residue $\tilde{x} \in \{0, \ldots, p^m - 1\}$ such that $\tilde{x} \equiv x \pmod{p^m\mathbb{Z}_p}$. It follows that $\tilde{x}+p^m \equiv x \pmod{p^m\mathbb{Z}_p}$, and thus $x \in B(\tilde{x}+p^m, p^{-m})$. But

$$p^{-m} \leq p^{-m_1} < \epsilon$$

and

$$\tilde{x} + p^m \geq p^{m_2} \geq N.$$

So $\tilde{x} + p^m$ is a natural number greater than or equal to N which is at most ϵ away from x. Since ϵ and x were arbitrary, we conclude that $\{k \in \mathbb{N} \mid k \geq N\}$ is dense in $\mathbb{Z}_p$. ■

Using this fact, we can show that a dense subset of $\mathbb{Z}_p$ gets mapped by $n \mapsto n!$ arbitrarily close to zero. Continuity handles the rest.

Proposition 3. *The map $n \mapsto n!$ does not admit a continuous extension $\mathbb{Z}_p \to \mathbb{C}_p$.*

Proof. Suppose $f\colon \mathbb{Z}_p \to \mathbb{C}_p$ is a continuous extension of $n \mapsto n!$. Fix $\epsilon > 0$ and $m \in \mathbb{N}$ so that $|p^m| < \epsilon$. Fix an arbitrary integer $n \geq p^m$. Then by expanding the factorial recurrence $n! = n(n-1)!$, we have

$$|f(n)| = \left|(p^m - 1)! \cdot p^m \cdot \prod_{k=p^m+1}^{n} k\right|$$

Since every term in this product is in $\mathbb{N}$, we have

$$|f(n)| \leq |p^m| < \epsilon.$$

By the continuity of f, we can fix a δ_n such that all $x \in B(n, \delta_n)$ satisfy

$$|f(x) - f(n)| < \epsilon,$$

which implies by the strong triangle inequality that $|f(x)| < \epsilon$. Since n was arbitrary, this holds for all $n \geq p^m$. However, by Lemma 1, every element $x \in \mathbb{Z}_p$ is in a ball $B(n, \delta_n)$. Thus $|f(x)| < \epsilon$ for all $x \in \mathbb{Z}_p$. Since $\epsilon > 0$ was arbitrary, it follows that $f(x) = 0$ for all $x \in \mathbb{Z}_p$. Since $n \mapsto n!$ is not the zero function on $\mathbb{N}$, this is contradictory. We conclude that $n \mapsto n!$ cannot admit a continuous extension $\mathbb{Z}_p \to \mathbb{C}_p$. ■

If we could ensure our function only included the p-adic units $\mathbb{Z}_p^\times$, then this issue would not arise. In 1975, Morita [6] apparently noticed this and defined his analogue to the Γ-function as a "restricted factorial" (nomenclature from Robert [7]) which does not include powers of p:

$$n!^* := \prod_{\substack{1 \leq k \leq n \\ (k,p)=1}} k.$$

Morita then shows that $(-1)^n(n-1)!^*$ can be extended to a continuous function $\mathbb{Z}_p \to \mathbb{Z}_p$.

While Morita himself referrde to $(n-1)!^*$ as the "gamma function," as early as 1981 with Barsky [1], authors have referred to the $\mathbb{Z}_p$ extension as Γ_p. This is so that Γ_p is a continuous function $\mathbb{Z}_p \to \mathbb{Z}_p$. We will use the modern notation, but citing Morita's results accordingly.

Definition 2. Define $\mathcal{N} := \{n \in \mathbb{N} \mid n \geq 2\}$. Fix p prime. Then define the *Morita Γ-function* $\Gamma_p \colon \mathcal{N} \to \mathbb{Z}_p^\times$ by

$$\Gamma_p(n) = (-1)^n(n-1)!^*,$$

for all $n \in \mathbb{N}$. Since the restricted factorial $n!^* \in \mathbb{Z}_p^\times$, we have $\Gamma_p(n) \in \mathbb{Z}_p^\times$ also.

To show that Γ_p has a continuous extension $\mathbb{Z}_p \to \mathbb{C}_p$, Morita showed that Γ_p is uniformly continuous on $\mathcal{N}$, viewed as a dense subset of $\mathbb{Z}_p$. This logic follows from the following result on interpolation.

Lemma 2 (5.10.1 in Gouvêa [5])**.** *Suppose S is a dense subset of $\mathbb{Z}_p$. If a function $f \colon S \to \mathbb{Q}_p$ is uniformly continuous and bounded with respect to the topology on $\mathbb{Q}_p$, then there exists a continuous function $\tilde{f} \colon \mathbb{Z}_p \to \mathbb{Q}_p$ such that $\tilde{f}(n) = f(n)$ for all $n \in \mathbb{N}$.*

Theorem 2 (Morita [6])**.** *The function Γ_p admits a continuous extension $\mathbb{Z}_p \to \mathbb{Z}_p$.*

Proof. We proceed by showing Γ_p is uniformly continuous on $\mathcal{N} \subseteq \mathbb{Z}_p$, thus proving Γ_p has a continuous extension $\mathbb{Z}_p \to \mathbb{Q}_p$ by Lemma 1 and Lemma 2. We then show that Γ_p has an image in $\mathbb{Z}_p$.

To start the former, let $\epsilon > 0$ be arbitrary. Since the valuation image of $\mathbb{Z}_p$ is $\mathbb{N}$, there is a minimal $m \in \mathbb{N}$ such that $|p^m| < \epsilon$. Let $n_0 \in \mathcal{N}$ be arbitrary and fix $n \in B(n_0, \epsilon) \cap \mathcal{N}$; that is, $n \equiv n_0 \pmod{p^m\mathbb{Z}_p}$. It suffices to show $\Gamma_p(n) \equiv \Gamma_p(n_0) \pmod{p^m\mathbb{Z}_p}$ also.

Without loss of generality, assume $n \geq n_0$, so $n = n_0 + \ell p^m$ for some $\ell \in \mathbb{N}$. For notational convenience, let $\prod^*$ denote the product on numbers coprime to p. By Definition 2, we have

$$\begin{aligned}
\Gamma_p(n)/\Gamma_p(n_0) &= \Gamma_p(n_0 + \ell p^m)/\Gamma_p(n_0) \\
&= \frac{(-1)^{n_0+\ell p^m}(n_0 - 1 + \ell p^m)!^*}{(-1)^{n_0}(n_0-1)!^*} \\
&= (-1)^{\ell p^m} \prod_{k=n_0}^{n_0+\ell p^m}{}^{*} k;
\end{aligned}$$

here we can split up the product into ℓ distinct products, and reindexing each to start at n_0:

$$\begin{aligned}
&= (-1)^{\ell p^m} \left(\prod_{k=n_0}^{n_0+p^m}{}^{*} k \right) \cdots \left(\prod_{k=n_0}^{n_0+p^m}{}^{*} k + (\ell-1)p^m \right) \\
&\equiv (-1)^{\ell p^m} \left(\prod_{k=n_0}^{n_0+p^m}{}^{*} k \right)^{\ell} \pmod{p^m\mathbb{Z}_p} \\
&\equiv (-1)^{\ell p^m} \left(\prod_{k=0}^{p^m}{}^{*} k \right)^{\ell} \pmod{p^m\mathbb{Z}_p};
\end{aligned}$$

finally, Morita uses a Theorem given by Gauss [4, no. 78], to write

$$\begin{aligned}
&\equiv (-1)^{\ell p^m}(-1)^{\ell p^m} \pmod{p^m\mathbb{Z}_p} \\
&\equiv 1 \pmod{p^m\mathbb{Z}_p}.
\end{aligned}$$

Thus we have $\Gamma_p(n) \equiv \Gamma_p(n_0) \pmod{p^m\mathbb{Z}_p}$, as desired. Since Γ_p is continuous on $\mathcal{N}$, a dense subset by Lemma 1, and bounded by Definition 2, Γ_p has a continuous extension $\mathbb{Z}_p \to \mathbb{Q}_p$ by Lemma 2. In a slight abuse of notation, we will call this extension Γ_p also.

It remains to be shown that $\Gamma_p(\mathbb{Z}_p) \subseteq \mathbb{Z}_p$. Fix $x \in \mathbb{Z}_p$. By the continuity of Γ_p there exists a $\delta > 0$ such that $\Gamma_p(B(x,\delta) \cap \mathbb{Z}_p) \subseteq B(\Gamma_p(x), 1)$. Since $\mathcal{N}$ is dense in $\mathbb{Z}_p$, there exists an $n \in \mathcal{N} \cap B(x,\delta) \cap \mathcal{N}$, and thus $|\Gamma_p(x) - \Gamma_p(n)| < 1$. However, by Definition 2, $|\Gamma_p(n)| = 1$, so by the strong triangle inequality we have

$$|\Gamma_p(x) - \Gamma_p(n) + \Gamma_p(n)| = |\Gamma_p(x)| \leq 1,$$

proving that $\Gamma_p(x) \in \mathbb{Z}_p$.

■

Inspired by Robert [7], we may define a function h_p to obtain a p-adic analogue to Theorem 1.

Proposition 4. *Define for all $x \in \mathbb{Z}_p$,*

$$h_p(x) = \begin{cases} -x & \text{if } x \in \mathbb{Z}_p^\times \\ -1 & \text{if } x \in p\mathbb{Z}_p \end{cases}$$

Then

$$\Gamma_p(x+1) = h_p(x)\Gamma_p(x)$$

for all $x \in \mathbb{Z}_p$.

Proof. Fix $x \in \mathbb{Z}_p$ and let $\epsilon > 0$ be arbitrary. Notice that the relation $\Gamma_p(n+1) = h_p(n)\Gamma_p(n)$ holds for $n \in \mathcal{N}$ by Definition 2 (and the definition of the restricted factorial).

We first consider the case where $x \in \mathbb{Z}_p^\times$, notice the function h_p restricted to $\mathbb{Z}_p^\times$ is a polynomial, which is continuous with respect to $\mathbb{Z}_p$. Γ_p is also continuous, and thus there exists a $\delta_1 > 0$ such that if $y \in \mathbb{Z}_p^\times$ and $|x - y| < \delta_1$, then $|h_p(x)\Gamma_p(x) - h_p(y)\Gamma_p(y)| < \epsilon$. There also exists a $\delta_2 > 0$ such that if $|(x+1) - (y+1)| = |x - y| < \delta_2$, then $|\Gamma(x+1) - \Gamma(y+1)| < \epsilon$. Since $\mathcal{N}$ is dense in $\mathbb{Z}_p$, we may fix an $n \in \mathcal{N}$ such that $|x - n| < \min\{1, \delta_1, \delta_2\}$.

Notice that $n \in \mathbb{Z}_p^\times$ since $|x| = 1$, $|x - n| \neq 1$, and all non-archimedean triangles are isosceles. Hence $|h_p(x)\Gamma_p(x) - h_p(n)\Gamma_p(n)| < \epsilon$ and $|\Gamma_p(x+1) - \Gamma_p(n+1)| < \epsilon$. By the strong triangle inequality, this implies

$$|(h_p(x)\Gamma_p(x) - \Gamma_p(x+1)) - (h_p(n)\Gamma_p(n) - \Gamma_p(n+1))| < \epsilon,$$

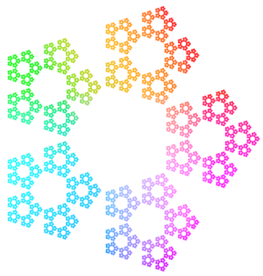
but $h_p(n)\Gamma_p(n) - \Gamma_p(n+1) = 0$, so $|h_p(x)\Gamma_p(x) - \Gamma_p(x+1)| < \epsilon$. Since $\epsilon > 0$ was arbitrary, it follows that $h_p(x)\Gamma_p(x) - \Gamma_p(x+1) = 0$, proving our claim.

A similar argument applies for when $x \in p\mathbb{Z}_p$. ■

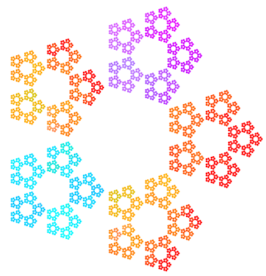
Now that we have proven Γ_p admits a continuous extension on the p-adic integers, it is sound to use our visualization method in $\mathbb{Z}_p$. We will use this to conceptualize an analogous result to the Legendre relation (Proposition 2) in $\mathbb{Z}_p$

4 Visualizing Γ_p

Let us now visualize the Morita Γ-function on the p-adic integers. We begin by plotting Γ_p using the standard identity plane for $p = 5, 7$ and 11, seen in Figure 4. Striking! The first noticable feature of these plots is the grouping of the colors in the transformation plane. It appears that the cosets $k + p\mathbb{Z}_p$ assume the colors of $\Gamma_p(k) + p\mathbb{Z}_p$ on the identity

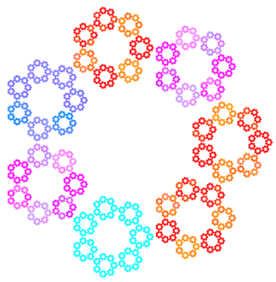


(a) $x \mapsto x$

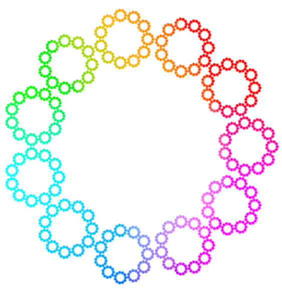


(b) $x \mapsto \Gamma_5(x)$

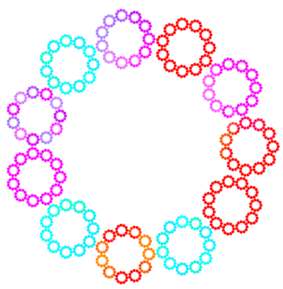
(c) $x \mapsto x$



(d) $x \mapsto \Gamma_7(x)$

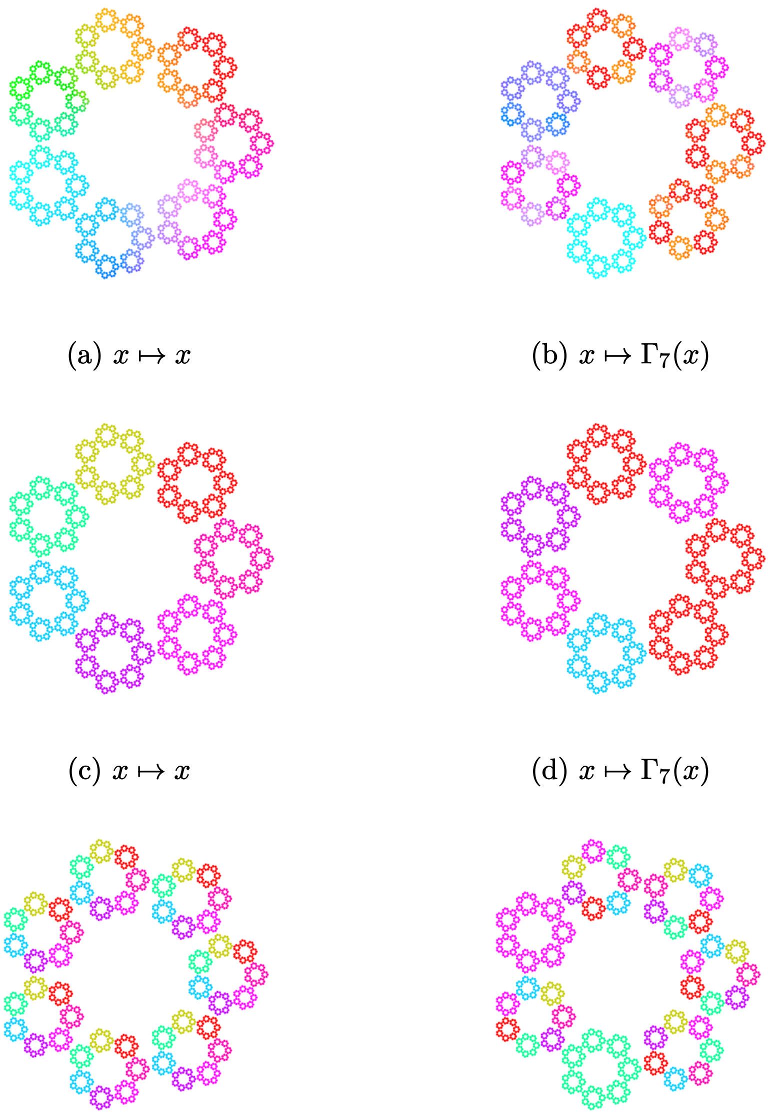


(e) $x \mapsto x$



(f) $x \mapsto \Gamma_{11}(x)$

Figure 4: Two-plane plots of Γ_p for $p = 5, 7$, and 11, using the standard identity plane at resolutions 5, 4, and 3, respectively.



(a) $x \mapsto x$

(b) $x \mapsto \Gamma_7(x)$

(c) $x \mapsto x$

(d) $x \mapsto \Gamma_7(x)$

(e) $x \mapsto x$

(f) $x \mapsto \Gamma_7(x)$

Figure 5: Two-plane plots of Γ_7 using the standard identity plane, highlighting cosets mod $7\mathbb{Z}_7$, and highlighting cosets mod $49\mathbb{Z}_7$.

k	$(1-k)$	$\Gamma_7(k)$	$\Gamma_7(1-k)$	$\Gamma_7(k)\Gamma_7(1-k)$
0	1	1	-1	-1
1	0	-1	1	-1
2	-1	1	1	1
3	-2	5	4	-1
4	-3	6	6	1
5	-4	4	5	-1
6	-5	1	1	1

Table 1: Coset analysis of $\Gamma_7(x)\Gamma_7(1-x)$. All values are mod $7\mathbb{Z}_7$.

plane. This is a consequence of our uniform continuity proof in *Theorem* 2 requiring the same ϵ for Γ_p's domain and image.

In search of an analogue of Proposition 2, let's take a closer look at Γ_7. In Figure 5, we've plotted Γ_7 with the standard plane in addition to highlighting the structure of its cosets mod $7\mathbb{Z}_7$ and $49\mathbb{Z}_7$. Considering the product $\Gamma_7(x)\Gamma_7(1-x)$, we use (c) and (d) in Figure 5 to generate Table 1.

At least on the level of cosets mod $7\mathbb{Z}_7$, there is clearly a pattern of alternating 1's and -1's, starting at $1+7\mathbb{Z}_7$. You may use (e) and (f) in Figure 5 to verify a few similar calculations mod $49\mathbb{Z}_7$. This provides a good guess for an analogue to the Legendre relation, which turns out to be true in general.

Proposition 5. *If $p \neq 2$, then Γ_p satisfies an analogue of the Legendre relation*

$$\Gamma_p(x)\Gamma_p(1-x) = (-1)^{r(x)}.$$

If $p = 2$, then $r(x)$ maps x to

Proof. c.f. Robert [7, p. 369], though we proceed by induction. Notice first that $\Gamma_p(1) = \frac{\Gamma_p(2)}{h_p(1)} = -1$. Similarly $\Gamma_p(0) = \frac{\Gamma_p(1)}{h_p(0)} = 1$. Thus $\Gamma_p(1)\Gamma_p(0) = -1$, and 1 is the smallest positive representative mod $p\mathbb{Z}_p$ of 1 for all primes p. Now suppose for some $n \in \mathbb{N}$ that

$$\Gamma_p(k)\Gamma_p(1-k) = (-1)^{r(k)},$$

for all $1 \leq k \leq n$. Then

$$\begin{aligned}\Gamma(n+1)\Gamma(-n) &= (h_p(n)\Gamma(n))\frac{\Gamma(1-n)}{h_p(-n)} \\ &= \frac{h_p(n)}{h_p(-n)}\Gamma(n)\Gamma(1-n) \\ &= \frac{h_p(n)}{h_p(-n)}(-1)^{r(n)},\end{aligned}$$

If $n \in \mathbb{Z}_p^\times$, then $h_p(n)/h_p(-n) = n/(-n) = -1$, but if $n \in p\mathbb{Z}_p$, then $h_p(n)/h_p(-n) = (-1)/(-1) = 1$. Thus, if $r(n) = p$, we have $\Gamma(n+1)\Gamma(-n) = (-1)^{r(n)}$, and if $r(n) \neq p$, then $\Gamma(n+1)\Gamma(-n) = (-1)^{r(n)+1}$. In either case, we have $\Gamma(n+1)\Gamma(-n) = (-1)^{r(n+1)}$, since $p \equiv 1 \pmod 2$. By induction, we conclude that $\Gamma_p(n)\Gamma_p(1-n) = (-1)^{r(n)}$ for all $n \geq 1$.

By a similar argument to Proposition 4, the density of $\{n \in \mathbb{N} \mid n \geq 1\}$ in $\mathbb{Z}_p$ and the continuity of Γ_p implies this identity extends to all of $\mathbb{Z}_p$. ■

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